SIMON FRASER UNIVERSITY
Faculty of Business Administration

FINAL EXAM

BUS 411 Fixed Income Security Analysis 23-3
Prof. Geoffrey Poitras

Academic Honesty: This assignment is individual work. Students are required to follow requirements of S10.01, especially Appendix A (see class web page for link).

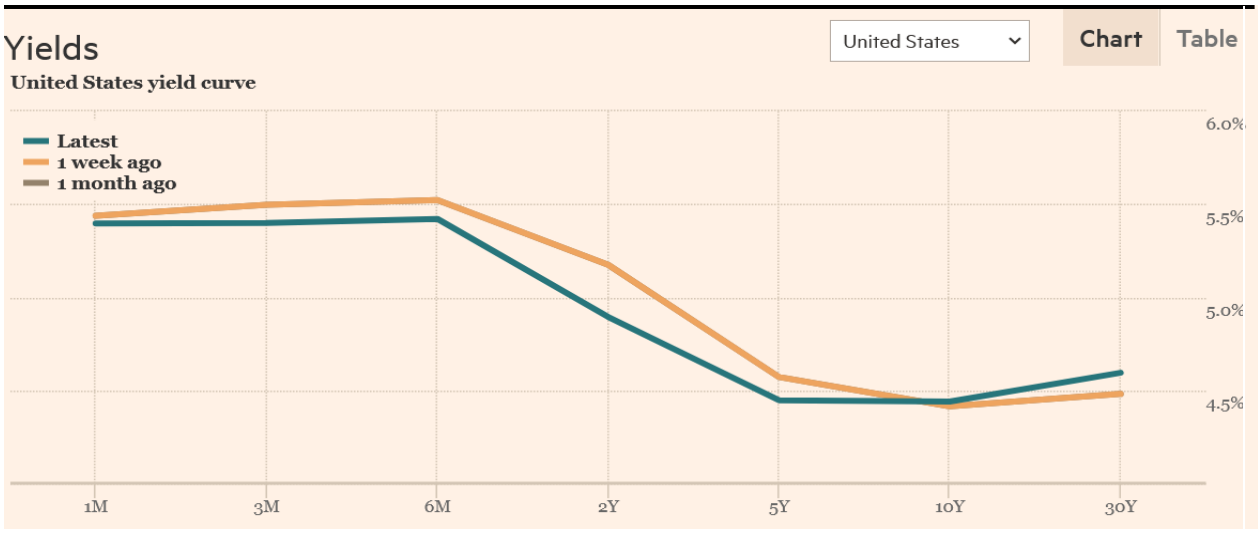
Rules for Submitting Final Exam: Exam is due in my email (poitras9@sfu.ca) no later than 8:30AM on Fri. Dec. 8, 2023. Late assignments will be assessed a 2% reduction per hour or part thereof, e.g., 11.5 hours late will have a 24% reduction.

DO ALL PARTS OF ALL QUESTIONS: Each question is worth 25 total points – for questions with two parts 15 points for part a) and 10 points for part b). Where applicable, provide calculations or code used to determine answers.

1. For the "Latest" Yield curve in the Chart below:

a) Calculate all annual implied zero coupon interest rates (z_t yearly) for the 1 to 10 year yield curve

b) With these z_t calculate a set of one year ahead implied forward rates (${}_t f_{t+1}$)



(Hint: Assume that the chart is for annual coupon par bond yields and that the 1 year yield is for a zero coupon.)

2a) Adam Smith provides the following description of the Million £ Loan:

> During the reigns of king William and queen Anne, large sums were frequently borrowed upon annuities for terms of years, which were sometimes longer and sometimes shorter. In 1693, an act was passed for borrowing one million upon an annuity of fourteen per cent., or of 140,000 l. a year, for sixteen years. In 1691, an act was passed for borrowing a million upon annuities for lives, upon terms which in the present times would appear very advantageous. But the subscription was not filled up. In the following year the deficiency was made good by borrowing upon annuities for lives at fourteen per cent., or at little more than seven years' purchase. In 1695, the persons who had purchased those annuities were allowed to exchange them for others of ninety-six years, upon paying into the Exchequer sixty-three pounds in the hundred; that is, the difference between fourteen per cent. for life, and fourteen per cent. for ninety-six years, was sold for sixty-three pounds, or for four and a half years' purchase. Such was the supposed instability of government, that even these terms procured few purchasers. In the reign of queen Anne, money was upon different occasions borrowed both upon annuities for lives, and upon annuities for terms of thirty-two, of eighty-nine, of ninety-eight, and of ninety-nine years.

i) Assuming arithmetically declining survival rates (uniform death rates), a maximum possible age of 86 and a market interest rate of 8%, what is the 'breakeven age' for a purchaser of the million £ loan life annuity.
ii) Explain how Smith arrives at the solution of 4.5 years purchase for the exchange of the life annuity for the term annuity of 96 years at the rate of £ 63 of term annuity for £100 of life annuity.
iii) Assuming arithmetically declining survival rates (uniform death rates) and a maximum possible age of 86, if the life annuity nominee is 12 years old, what is the 'true yield' for the life annuity?

b) i) On Nov. 18, 2023 the Cargojet 5.25% debenture maturing June 30,2026 was selling for $93.35 (flat), what is the promised yield to maturity for this bond? On the same day, the Slate Office REIT 7.5% debenture maturing Dec. 31, 2027 was selling for $57.50 (flat), what is the yield to maturity for this bond?

ii) Using the par bond yield curve from Question #1, construct a portfolio of a 2 year + 30 year bond that has the same modified duration as the ten year bond. What are the convexities and 'time value' for the 2 + 30 year portfolio and the 10 year bond?

iii) Witt (1613) poses the following problem: A oweth to B £1200 to be paid in 6 yeares, in 12 equal payments, viz. at the end of each halfe yeare £100. They agree to cleare this debt in 3 yeares, in 6 equall payments, viz. at the end of each halfe yeare, one payment. The Question is, what each payment ought to be, reckoning interest after the rate of 10 per cent per Ann. and int. upon int.?

iv) Halley (1698) poses the following problem: An annuity of £20 being in possession for the term of 21 years, and for £40 paid down it can be prolonged for 10 years more to 31 years; what is the rate of interest required? (Hint: Evaluate the interest rate associated with paying £40 today for a cash flow of £20 that will last for 10 years after the 21 year annuity reaches maturity.)

3. Using the CME price quotes in the APPENDIX solve for the 3 and 6 month foreign interest rates implied by Covered Interest Parity using the SOFR futures and CME currency futures for the A$ and £. (Hint: See the worked example on the class webpage; use the nearby contract for the spot price; final answers to be expressed as annualized interest rates.)

4. (Show Calculations, attach spreadsheet or mathematica code used to solve the problem)

The Canada Pension Plan (Canada Pension Plan, RSC 1985, c. C-8) is a complicated defined benefit plan that has several provisions, such as adjustments for 'drop-out years', that impact the required number of contribution years required to receive the maximum pension payment. In addition, the plan allows for pension payments to be started at any time between age 60 and age 70, with appropriate adjustment in the payment amount. For a male age 65 that is ineligible for 'drop-out provisions' that reduce the number of maximum contribution years to receive the full benefit, 39 qualifying years of maximum contributions is required for the full benefit of $15,678.84 per year ($1,306.57/mo.).

Using the maximum contribution amounts from the following Table and assuming investment returns applicable for each of the following intervals:

1984-1990	10.5%
1991-2000	7.5%
2001-2010	4.5%
2011-2022	2.5%

i) For both the employee and self-employed, calculate the amount that would have been earned (from 1984 to 2022) if the CPP contributions given in the Table below had been invested instead of being paid into CPP. (Hint: The calculation involves starting from 1984 and accumulating investment in a fund to arrive at a final total in 2022.)

Using the calculation from a) for both an employee and a self-employed individual that has made the maximum contributions detailed in the Table, solve for the following:

ii) Assuming arithmetically declining survival rates and a maximum possible age of 95, calculate the implied interest for this individual electing to receive the maximum CPP pension payment at age 65 of $15,678.84 per year ($1,306.57/mo.). (Hint: This is the same type of calculation as that for Assignment #1, 2b)

iii) For both the employee and self-employed, if the individual opts to defer taking the CPP until age 70 when the annual payment would be $1855.00/month; $22,260 annual (no further CPP contributions are required or made after 2022), calculate the implied interest rate assuming arithmetically declining survival rates and a maximum possible age of 95.

iv) At a current interest rate of 5.5% what is the breakeven age at which deferring CPP until age 70 has the same present value as taking CPP at age 65. (Hint: This involves using the cash flows from b) and c) above and doing a expected present value calculation for age 65.) How does your answer change if the interest rate is 7.5%?

Table: Maximum Canada Pension Plan Contributions, 1984-2022

	Employee Contribution	Self-Employment Contribution
1984	$338.40	$676.80
1985	$379.80	$759.60
1986	$419.40	$838.80
1987	$444.60	$889.20
1988	$478.00	$956.00
1989	$525.00	$1050.00
1990	$574.20	$1148.40
1991	$632.50	$1263.00
1992	$696.00	$1392.00
1993	$752.50	$1505.00
1994	$806.00	$1612.00
1995	$850.50	$1701.00
1996	$925.82	$1851.62
1997	$993.22	$1996.44
1998	$1,068.80	$2,137.6
1999	$1,186.50	$2,373.00
2000	$1,329.90	$2,659.80
2001	$1,496.40	$2,992.80
2002	$1,673.20	$3,346.40
2003	$1,801.80	$3603.60
2004	$1,831.50	$3663.00
2005	$1,861.20	$3,722.40
2006	$1,910.70	$3821.40
2007	$1,989.90	$3979.80
2008	$2,049.30	$4,098.60
2009	$2,118.60	$4,337.20
2010	$2,163.15	$4,236.20
2011	$2,217.60	$4,435.20
2012	$2,306.70	$4,613.40
2013	$2,356.20	$4,712.40
2014	$2,425.50	$4,851.00
2015	$2,479.95	$4,959.90
2016	$2,544.30	$5,088.60
2017	$2,564.10	$5,128.20
2018	$2,593.80	$5,187.60
2019	$2,748.05	$5,497.80
2020	$2,898.00	$5,796.00
2021	$3,166.45	$6,332.90
2022	$3,499.80	$6,999.60

https://www.canada.ca/en/revenue-agency/services/tax/businesses/topics/payroll/payroll-deductions-contributions/canada-pension-plan-cpp/cpp-contribution-rates-maximums-exemptions.html

APPENDIX FOR QUESTION #3 Quotes for Nov. 18, 2023

AUSTRALIAN DOLLAR FUTURES - SETTLEMENTS

MONTH	OPEN	HIGH	LOW	LAST	CHANGE	SETTLE	EST. VOLUME	PRIOR DAY OI
DEC 23	.64765	.65225	.64580	.65220	+.00385	.65155	83,240	192,530
JAN 24	.64825	.65280	.64645A	.65265A	+.00385	.65215	49	222
FEB 24	-	.65260B	.64710A	.64710A	+.00385	.65280	0	12
MAR 24	.64930	.65390	.64755A	.65375A	+.00385	.65325	128	978
APR 24	-	-	-	-	+.00390	.65370	0	0
JUN 24	.65250	.65470B	.64905A	.65265A	+.00400	.65460	8	111
SEP 24	-	-	-	-	+.00400	.65550	0	14
DEC 24	-	.65585B	.65050A	.65050A	+.00400	.65580	0	62

BRITISH POUND FUTURES - SETTLEMENTS

MONTH	OPEN	HIGH	LOW	LAST	CHANGE	SETTLE	EST. VOLUME	PRIOR DAY OI
DEC 23	1.2416	1.2468	1.2376	1.2462A	+.0031	1.2451	93,932	205,632
JAN 24	1.2417	1.2470	1.2381A	1.2466A	+.0031	1.2454	126	587
FEB 24	1.2411	1.2472B	1.2384A	1.2472B	+.0031	1.2457	309	334
MAR 24	1.2412	1.2474B	1.2386A	1.2474B	+.0031	1.2459	129	2,773
APR 24	-	-	-	-	+.0031	1.2461	0	0
JUN 24	1.2456	1.2456	1.2393A	1.2424A	+.0032	1.2466	10	299
SEP 24	-	1.2483B	1.2402A	1.2402A	+.0033	1.2472	0	213
DEC 24	-	1.2487B	1.2409A	1.2409A	+.0033	1.2476	0	17

THREE-MONTH SOFR FUTURES - SETTLEMENTS

MONTH	OPEN	HIGH	LOW	LAST	CHANGE	SETTLE	EST. VOLUME	PRIOR DAY OI
SEP 23	94.6525	94.6525	94.6475	94.6525	+.0025	94.6525	40,910	949,848
OCT 23	94.6350	94.6350	94.6350	94.6350	.0000	94.6400	32	4,335
NOV 23	94.6275	94.6300B	94.6275	94.6300B	.0000	94.6325	130	2,840
DEC 23	94.6300	94.6325	94.6200	94.6225	-.0050	94.6225	241,830	1,445,328
JAN 24	-	-	-	-	-.0050	94.6400	0	2,514
FEB 24	-	94.7000B	94.6850A	94.7000B	-.0050	94.6850	0	1,311
MAR 24	94.7800	94.7950	94.7500	94.7650	-.0150	94.7600	292,288	1,126,461
APR 24	94.8550	94.8700B	94.8350A	94.8700B	-.0150	94.8400	5	420
MAY 24	-	94.9700B	94.9200A	94.9700B	-.0150	94.9300	0	0

BONUS (5 points)

You are in the market for a house. Your effective all-in market borrowing rate for a first mortgage with a 4 year term from a financial institution is 6.69% with a 30 year amortization period. The asking price on the house is $900,000. If you decide to take up the mortgage and have a 10% down payment, what will be the all-in borrowing rate if the Canada Guarantee mortgage insurance premium is assessed at 5% of the mortgage value?